On the Eisenstein ideal for imaginary quadratic fields - Erratum

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The statement in Theorem 15 about the perfectness of the Poincaré and Lefschetz pairings for Dedekind domains (attributed to Urban) is incorrect. The pairings are only perfect modulo torsion. Theorem 15 was only used in the proof of Lemma 16 for proving that the image of the restriction map to the boundary fills out half the size of the cohomology group of the boundary with coefficients in $R/\mathfrak{m}^r$.

The proof of Lemma 16 can be corrected as follows:

Put $X = \Gamma\backslash\overline{\mathbf{H}}_3$, let R be a complete discrete valuation ring with finite residue field of characteristic $p > 2$, $\mathfrak{m}$ its maximal ideal and L its field of fractions. Let ι be an orientation-reversing involution on X. Denote by a superscript $+$ (respectively $-$) the $+1$-(respectively -1-) eigenspace for the induced involutions on cohomology groups. Instead of the Poincaré duality we apply the Pontryagin duality for arithmetic groups (see Section 1.9 of [Hid93]). Let M be any R-module and write $M^* = \mathrm{Hom}(M, L/R)$ for its Pontryagin dual. Pontryagin duality combined with the boundary exact sequence of homology and cohomology groups and the Lefschetz duality of [Gre67] (28.18) gives us the following diagram:

$$\begin{array}{ccccc}
H^2_c(X,M)^{\pm} & \times & H^1(X,M^*)^{\mp} & \longrightarrow & L/R \\
\uparrow{\scriptstyle \partial} & & \downarrow{\scriptstyle \mathrm{res}} & & \\
H^1(\partial X,M)^{\pm} & \times & H^1(\partial X,M^*)^{\mp} & \longrightarrow & L/R \\
\uparrow{\scriptstyle \mathrm{res}} & & \downarrow{\scriptstyle \partial} & & \\
H^1(X,M)^{\pm} & \times & H^2_c(X,M^*)^{\mp} & \longrightarrow & L/R
\end{array}$$

Here the vertical sequences are exact, the horizontal pairings are perfect and res and ∂ are adjoint, i.e.

$$\langle \mathrm{res}(x), y\rangle = \langle x, \partial(y)\rangle$$

for all $x \in H^1(X,M), y \in H^1(\partial X, M^*)$. As the pairings are induced from the cup product and evaluation on the fundamental class (which is inverted by the involution ι) the $+1$- and -1-eigenspaces are dual to each other.

Now proceed as in the proof as in the paper, i.e. deduce from the adjointness of res and ∂ and the perfectness of the pairings that $\mathrm{im}(\mathrm{res})^\perp = \mathrm{im}(\mathrm{res})$ and so for $M = R/\mathfrak{m}^r$ that

$$\#\mathrm{im}(\mathrm{res}) = \frac{1}{2}\#H^1(\partial X, R/\mathfrak{m}^r).$$

References

[Hid93] H. Hida, p-ordinary cohomology groups for SL(2) over number fields, Duke Mathematical Journal Vol. 69, no. 2, 259-314.

[Gre67] M. Greenberg, Lectures on algebraic topology, W.A. Benjamin, New York, 1967.