Workshop on Bianchi and Siegel modular forms
Sheffield 14-16 July 2014
Abstracts

June 17, 2014

Siegfried Böcherer (Mannheim)
Congruence primes via higher L-functions
Using the doubling method, one can detect congruence primes for Siegel modular forms from the denominators of critical values for standard L-functions (see H.Katsurada, J.Brown and others). The aim of the talk is to explore similarly the meaning of denominators for triple product L-functions and for standard L-functions after twisting by Dirichlet characters.

Thanasis Bouganis (Durham)
On special values of L-functions attached to half-integral weight Siegel modular forms
In this talk we will discuss algebraic and p-adic properties of special values of L-functions attached to Siegel modular forms of half-integral weight. These L-values, quite similarly to the integral weight situation, can be studied using two different approaches. One is based on the doubling method and the other on the Rankin-Selberg method. In the integral weight situation there has been considerable work both with respect to the algebraicity of these special L-values (Sturm, Harris, Garrett, Shimura, Böcherer) and to the existence of p-adic measures (Panchishkin, Böcherer and Schmidt). In this talk we will consider the half-integral weight situation and discuss extensions of the above works in this direction.

Jim Brown (Clemson)
The Ikeda ideal and a conjecture of Katsurada
Let f be a newform of level 1 and weight $2k-n$ for k and n positive even integers. It is well known one can associate to f a Siegel modular form $\mathrm{Ik}(f) \in S_k(\mathrm{Sp}_{2n}(\mathbb{Z}))$ satisfying

$$L(s, \mathrm{Ik}(f); \mathrm{st}) = \zeta(s) \prod_{i=1}^{n} L(s+k-i, f).$$

The form $\mathrm{Ik}(f)$ is known as the Ikeda lift of f.
In this talk we discuss congruence primes for the Ikeda lift of f. In particular, we consider a conjecture of Katsurada stating that primes dividing certain L-values of f are congruence primes for the Ikeda lift. Katsurda has provided evidence for this conjecture by showing if a prime divides these L-values and some additional conditions are satisfied, then this prime is a congruence prime for $\mathrm{Ik}(f)$. In joint work with Rodney Keaton we define the Ikeda ideal and study this to produce a lower bound on the number of all congruences between the Ikeda lift of f and forms not lying in the space spanned by Ikeda lifts. We also show how this recovers Katsurada's result as a special case.
One would like to use such a result to produce Bloch-Kato type results for f. Since one does not currently know the existence of Galois representations for Siegel modular forms of genus larger than 2 this is not possible. However, it is interesting to note that the L-values that control the congruences for Ikeda lifts are given by $L(k, f) \prod_{i=1}^{\frac{n}{2}-1} L(2i+1, \mathrm{ad}^0 f)$ where previous results using Saito-Kurokawa lifts are governed by just the L-value $L(k, f)$. This suggests that while congruence primes for Saito-Kurokawa lifts are only seeing Bloch-Kato results for $\rho_{f,\ell}$, congruences for Ikeda lifts (accepting the existence of the conjectured Galois representations) see results for $\rho_{f,\ell}$ as well as the adjoint representations.

Lassina Dembélé (Warwick)
Theta lifts of torsion classes of Bianchi groups

Neil Dummigan (Sheffield)
Langlands functoriality and Harder's conjecture
We look at an instance of Langlands functoriality, lifting from $\mathrm{SO}(n+1, n)$ to $U(n, n)$, and apply the case $n = 2$ to turn Harder's conjectural congruence into a congruence between a Hermitian Klingen-Eisenstein series and a cusp form.

Kris Klosin (CUNY)
Congruences and $R = T$ theorems
We will report on a recent result (joint with T. Berger and K. Kramer) on measuring higher congruences among modular forms. We will discuss how to apply this result to the study of modularity of reducible residual Galois representations and their lifts to characteristic zero. As a consequence we will obtain a new modularity theorem for Galois representations over imaginary quadratic fields.

Jolanta Marzec (Bristol)
Non-vanishing of fundamental Fourier coefficients of Siegel modular forms
We are going to investigate Fourier coefficients of Siegel modular forms of degree 2. The ones of special interest are those determined by matrices with fundamental discriminant. Whenever we are able to say that one of them does not vanish, we get - through the generalized Böcherer's conjecture - the non-vanishing results for

central values of L-functions. We will discuss the cases of congruence subgroups $\Gamma_0(N)$ and $\Gamma^{\mathrm{para}}(N)$ with N square-free.

Ameya Pitale (Oklahoma)
Local and global Maass relations
Given an elliptic modular form of weight $2k-2$, k even, and full level, one can construct scalar valued holomorphic Siegel modular forms of weight k, called the Saito Kurokawa lifts. These lifts are also characterized as those Siegel modular forms whose Fourier coefficients satisfy certain recurrence relations – called the Maass relations. One of the proofs of the above goes via the theory of holomorphic Jacobi forms, which forms a bridge between the elliptic modular forms and Siegel modular forms. In this talk, I will present another approach to proving the Maass relations for Saito-Kurokawa lifts. Using the fact that the local representations corresponding to a Saito-Kurokawa lift F has "special Bessel models", one can rewrite the Fourier coefficients of F in terms of local Bessel functions. Then the global Maass relations reduce to certain recurrence relations for the local Bessel functions, which we call the "local Maass relations", and these are obtained using only the local representation theory. The advantage of this approach is that it is applicable to more general situations where other methods like the theory of Jacobi forms is not available. This is joint work with Abhishek Saha and Ralf Schmidt.

Alexander Rahm (Galway)
Bianchi modular forms of varying discriminant, level and weight
This is joint work with Mehmet Haluk Sengun. In our paper, we have presented vast computer calculations, locating several of the very rare instances of level one cuspidal Bianchi modular forms that are not lifts of elliptic modular forms. Meanwhile, we have moved on from level one - the full Bianchi groups - to congruence subgroups in the Bianchi groups. This allows us to push the calculations simultaneously into three dimensions - varying discriminant, level and weight. We have been granted 900,000 processor hours for this purpose, and the results will soon be available to the public on the LMFDB database, providing number theorists with a broad picture of the dimensions of many thousands of spaces of Bianchi modular forms, hopefully facilitating some progress towards an extension of the modularity theorem to the imaginary quadratic case.

Abhishek Saha (Bristol)
Siegel modular forms of degree 2: Fourier coefficients, L-functions, and functoriality
This will be for the most part a survey talk. I will explain the link between Siegel cusp forms of degree 2 and automorphic representations of GSp(4). I will talk about the significance of their Fourier coefficients, and describe several known

results and still unproven conjectures. Towards the end of the talk, I will briefly talk about lifts to and from spaces of Siegel modular forms, and in particular, outline a proof (via the converse theorem) of the functorial transfer for full level Siegel cusp forms to GL(4). This last result was proved in collaboration with Ameya Pitale and Ralf Schmidt.

Haluk Şengün (Warwick/Sheffield)

Cohomology of Bianchi Groups and Arithmetic

The recent years have witnessed significant developments in our understanding of the Langlands Programme for GL(2) over CM fields. In this survey talk, I will focus on the particular case of GL(2) over imaginary quadratic fields.

In this special case, the relevant modular groups are known as Bianchi Groups and their study goes back to 1890's. The talk will put the cohomology of Bianchi groups in the center and will consider its connections with arithmetic from the perspective of the Langlands Programme, discussing open problems and recent developments.

Jacques Tilouine (Paris)

Big Image of Galois representations and congruence ideals

In a joint work in progress with Hida, we define an invariant for "general" Hida families of automorphic forms on GSp_4 and we give a first result towards its relation with certain congruence ideals.

Lynne Walling (Bristol)

Diagonalising spaces of Siegel Eisenstein series

We introduce Siegel Eisenstein series, and without knowing their Fourier coefficients, analyse the action of the Hecke operators on a basis for the space. For square-free level we can diagonalise the space with respects to all the Hecke operators, and for arbitrary level we can diagonalise the space with respect to those Hecke operators attached to primes not dividing the level. We also consider Siegel Eisenstein series of half-integral weights.